Developing Bond Graph Models for a Simplified Formula 1 Car

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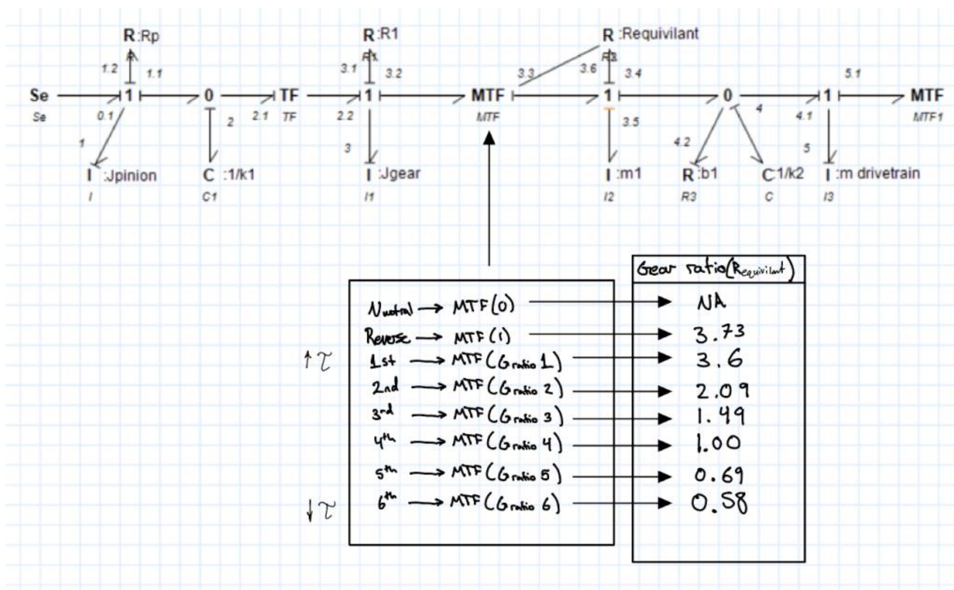
Introduction

Formula One racing is increasing in worldwide popularity every day. The talented drivers on challenging tracks create highly entertaining races. These exciting races would not be possible without the technically advanced high performing race cars. To gain the performance, each subsystem is highly optimized to perform in synergy with every other subsystem. Efficiency is important, but the balance of all systems is more important to make a winning car. To better understand the performance of an F1 car, subsystems were modeled using Bond Graph. This method reveals key factors that can hinder the system or create great variations. F1 races are extremely competitive and actual schematics of a vehicle are not readily available, so we based many of our models on road car components. While the intended purpose between a street legal car and an F1 car are vastly different it is assumed that these systems would be similar, or serve the similar function, with different performance specifications. Important systems included in this report are the transmission, brakes, suspension, radiator, energy recovery system, aerodynamic components, internal combustion engine, power steering, and turbocharger. Each section provides a brief overview of the subsystem to be modeled, an interpretation of the bond graph, linearization of the bond graph if needed, and equations to model the system.

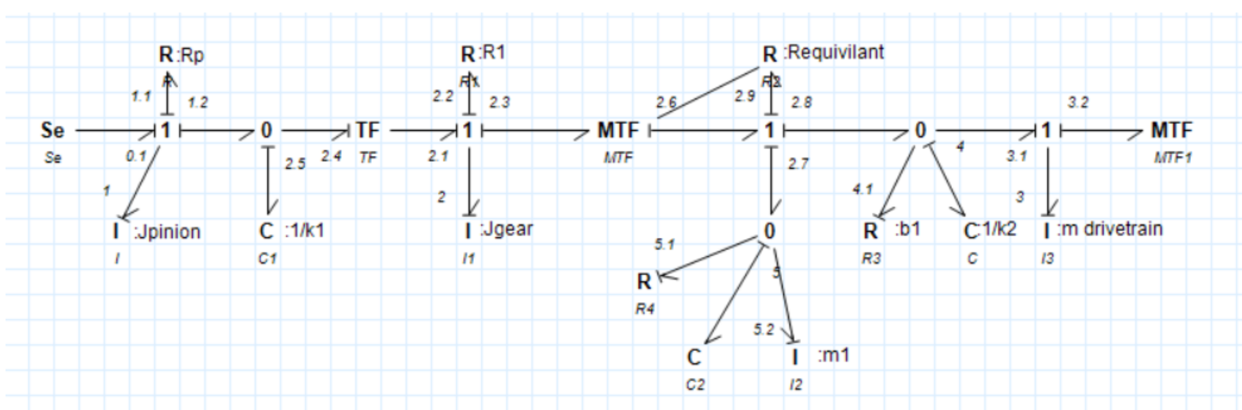
Transmission

Assumptions: The MTF acts as a multi selector tool controlled by the input of the gear shifter deciding the value of Requivalent.



The torque of the motor Se goes into the massive pinion, which has compliance C:1/k1 which goes to a transformer which is the radius of the pinion, which goes to the gear pack, which has J gear. This goes to an MTF, multiselector, which acts as a selector for the control to change gears. The gear ratio values are selected and put into the Requivilant Resistors exist naturally in the system. This then goes to the drive train which is a long, massive element which has inertance and stiffness effects. It ends in a differential, related the input theta from the drivetrain to the output thetas of the wheels, which is modeled elsewhere. The nonlinearities are shown by orange causal marks.

The linearization is shown below:



To linearize this model, the resistance and compliances of the output of the gearbox needed to be considered, which allowed for a linear system.

The final equations are shown below:

$$1: \theta_1 = \Sigma F(t) = \dot{\theta}_1 \cdot R_p + \ddot{\theta}_1 \cdot J_{pin}$$

$$1: \dot{\theta}_2 = \Sigma f(t) = \dot{\theta}_2 \cdot R_1 + \ddot{\theta}_2 \cdot J_{gear}$$

$$1: \dot{\theta}_3 = \Sigma f(t) = \dot{\theta}_3 \cdot R_{eq} + \ddot{\theta}_3 \cdot m_1$$

$$1: \dot{\theta}_4 = \Sigma f(t) = \ddot{\theta}_4 \cdot m_{dt}$$

$$1: \dot{\theta}_{3/4} = \Sigma f(t) = \dot{\theta}_{3/4} \cdot b_1 + \theta_{3/4} \cdot K_2$$

$$0: \tau_1 = \Sigma \dot{\theta}_1 = TF$$

$$0: \tau_2 = \Sigma \dot{\theta}_3 = \theta_{3/4} + \dot{\theta}_4$$

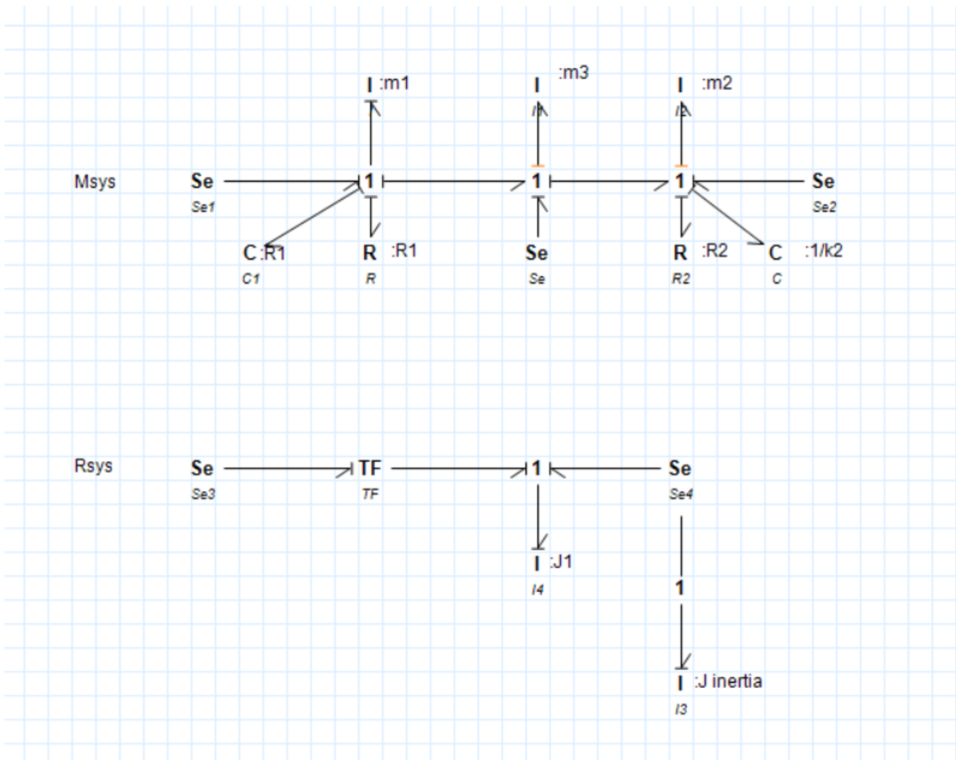
Sources Used: Balerna, Camillo, et al, Project Bobca

Brakes

Assumptions:

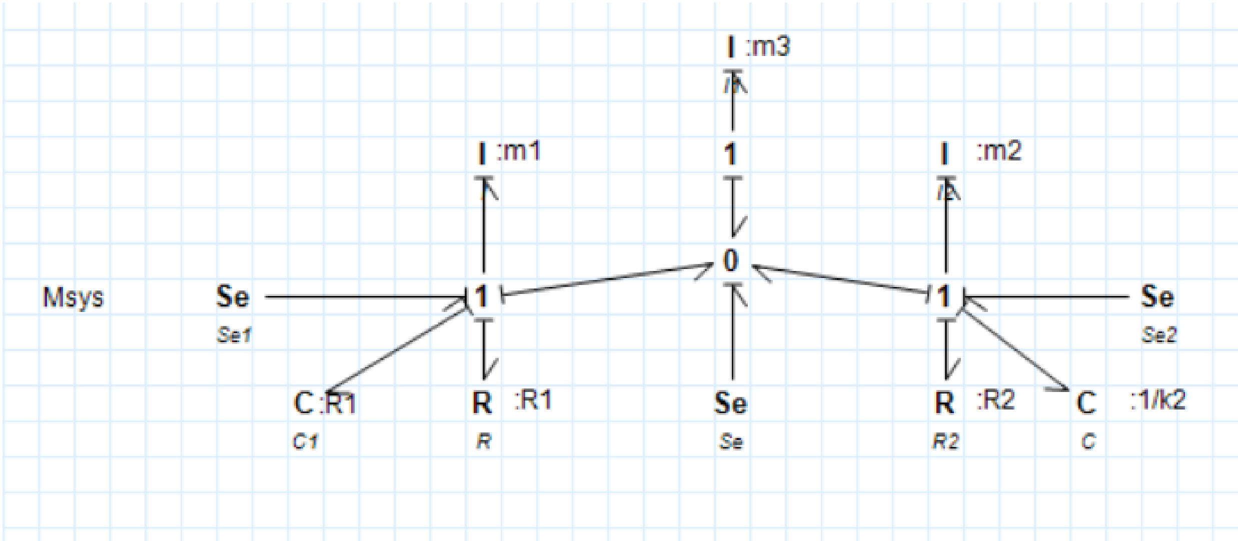
The key assumption is that this bond graph is a quarter of a car, only modeling one brake system. The system is assumed to be able to be broken up into multiple systems and combined later.

Bond graph with description:



The bond graph for the brake is split up into two separate models, the rotational and the mechanical systems (Rsys and Msys respectfully). Se1 is an electrical input that activates the brake pads m1 and m2, and presses into the rotor, m3. Brake pads, m1 and m2 have identical forces, Se1 and Se2 that press the brake pads into the rotor to slow the wheels, the output being force Se, attached to m3. In the rotational system, a force applied to the rotor, J1, is generated by the contact of the brake pads, Se3 and Se4. The nonlinearities are shown by orange causal marks. The redesigned linearized model is shown below:

In order to linearize the mechanical brake system, The break pads must be modeled as if they are in parallel. The final equations are shown below:



Mechanical

$$1 : \dot{x}_1 \left\{ P(t) = \ddot{x}_1 (m_1) + \dot{x}_1 \cdot R_1 + x \cdot K_1 \right.$$

$$1 : \dot{x}_2 \left\{ P(t) = \ddot{x}_2 (m_3) + F(t) \right.$$

$$1 : \dot{x}_3 \left\{ P(t) = \ddot{x} (m_2) + x \cdot k_2 + \dot{x} \cdot R_2 \right.$$

Rotational

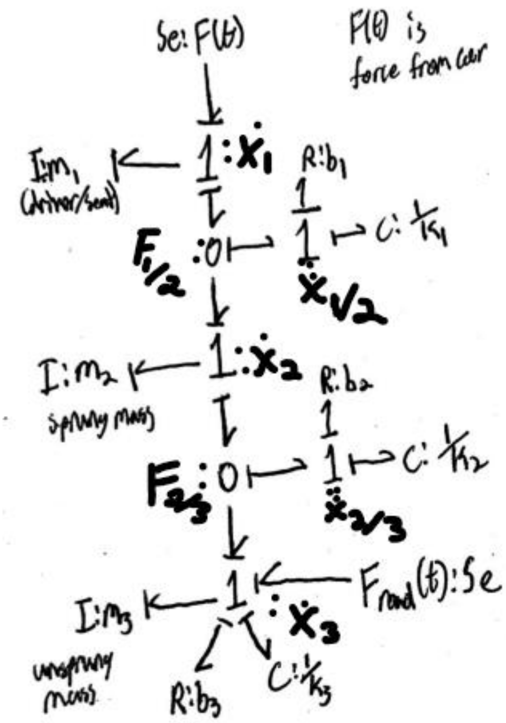
$$1 : \dot{\theta}_1 \left\{ f(t) = \ddot{\theta}_1 \cdot I : J_1 \right.$$

$$1 : \dot{\theta}_2 \left\{ F(t) = \ddot{\theta} \cdot I : J_i \right.$$

TF

Sources Used: (SAGE journals)

Quarter Car (Suspension)



The bond graph of the quarter car accounts for several masses, forces, resistances, and capacitances. The input force at the top of the system is from the car itself pressing into the system. The three masses on the left side are for the driver, sprung, and un-sprung masses. There are physical supports with compliance and resistance as well as the main compliance and resistance from the car's suspension spring and damper. The resistance, compliance, and mass at the bottom junction are associated with the tire. The mass of the tire is considerable, but it is not a rigid body and its walls/pressure has associated damping and capacitance. At the bottom is an input force that varies with respect to time to account for road/track conditions that could push into the tire or drop away from it. This model of the system is naturally linear while also factoring in different capacitances and resistances within the system.

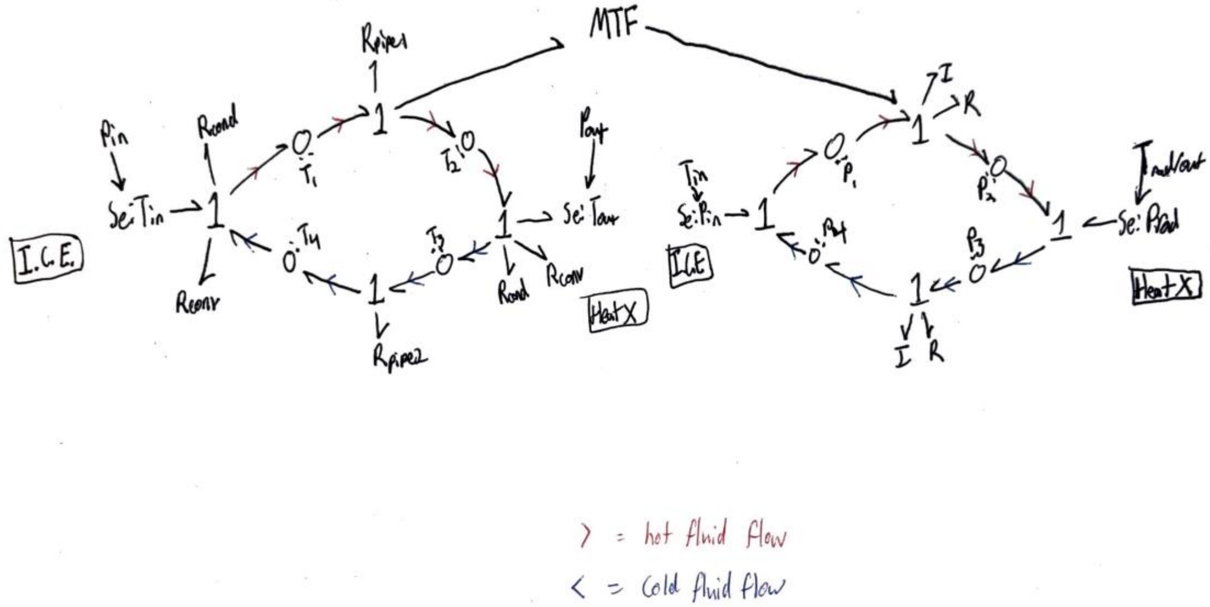
$$1: \dot{x}_1 : F(t) = m_1\ddot{x} + F_{1/2}$$
$$1: \dot{x}_{1/2} : F_{1/2} = b_1\dot{x}_{1/2} + k_1 x_{1/2}$$

$$1: \dot{x}_2 : F_{1/2} = m_2\ddot{x}_2 + F_{2/3}$$

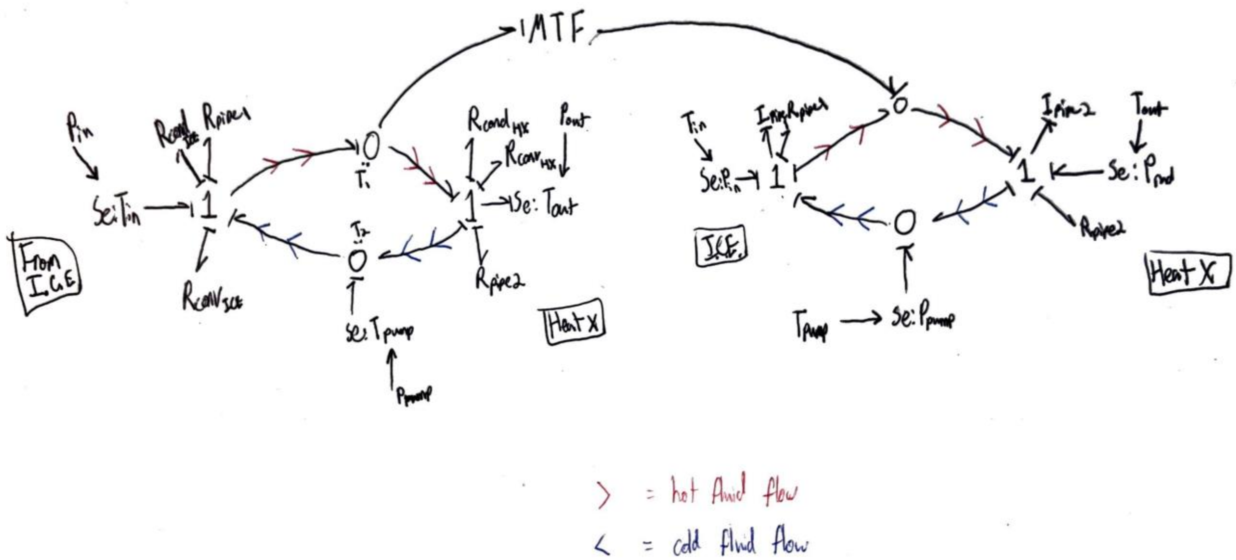
$$1: \dot{x}_{2/3} : F_{2/3} = b_2\dot{x}_{2/3} + k_2 x_{2/3}$$
$$1: \dot{x}_3 : F_{2/3} = m_3\ddot{x}_3 + b_3\dot{x}_3 + k_3 x_3 + F_{road}(t)$$

$$0: F_{1/2} : \dot{x}_1 = \dot{x}_2 + \dot{x}_{1/2}$$
$$0: F_{2/3} : \dot{x}_2 = \dot{x}_3 + \dot{x}_{2/3}$$

Radiator



This is one of the first iterations of the Radiator system. It contains several non-linearities and junctions that do not provide useful information.



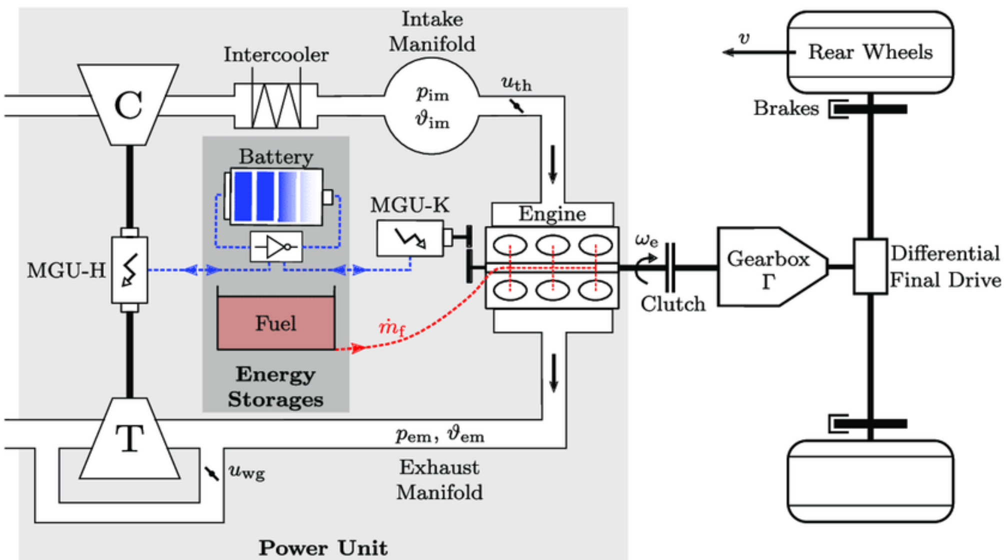
The Radiator system is decoupled into two graphs that separate temperature and pressure. Yet the two systems are important to each other and require input information from the other system. Each source has an information corresponding to a value found in the other graph, the heat source is dependent on the pressure source and vice versa. The Internal Combustion Engine, ICE, heats the coolant and pumps it to the radiator. At the radiator the coolant releases heat to the outside air and pumps it back to the ICE. The pipes of inertances and resistances that effect the flow and temperature of the coolant. This is especially important in the top part of the graph because the coolant is going to be at higher temperatures and will be releasing heat on the way to the radiator. The heat and possible cooling will have subsequent pressure changes. The MTF is included to relay the information on pressure and temperature effects between the two systems and handle changes between the two sources at either end.

Both halves of the system did require some consolidation and linearization. This was complicated by the MTF, inertances, and sources on either end of the graph. After condensing the top and bottom flows on either side, the bottom/ends still had a non-linearity and that that was addressed by adding a pump in parallel to the flow. The pump may not affect the entire flow but would help boost the pressure in the flow so that the coolant can continue to flow in the pipe to the ICE. This may be needed because at the cooler temperatures the coolant does not have as much presser and may need help reaching the ICE. The pump can also be seen in the temperature bond graph because a running pump is going to add heat to the system and it is important that heat is monitored. The point of the coolant is to remove heat that would inhibit the function of the ICE, if the assisting pump adds to much heat to the system then the coolant cannot properly cool the engine and it will fail. (Scarborough)

Energy Recover System (ERS)

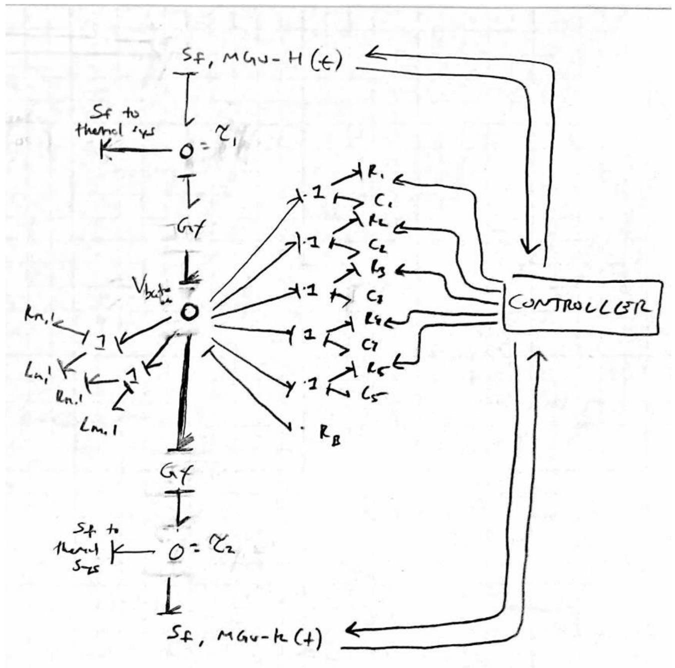
Assumptions: The pressure losses in the cooling of the ERS motor generators, and the fluidic system in general are negligible. A controller exists to control the voltages and manage when each ERS module is in generator or motor mode.

The principle of the energy recovery system on an F1 car is to capture losses from the engine, braking, and transmission system and generate energy that is stored in a battery. The controller can then decide when to utilize this energy to provide bonus power or reduce turbo lag. This is possible because the ERS system contains both an MGU-H (motor generator unit – heat) and MGU-K (motor generator unit – kinetic). The MGU-H sits in between the stages of the turbo, capturing energy when the turbo is spinning, and deploying it to reduced lag when the driver is accelerating. The MGU-K is attached to the transmission and captures energy when the driver is braking and deploys it back into the transmission for bonus power when the driver demands higher acceleration. Due to then high speeds of the MGU-H and MGU-K, an oil cooler is needed to keep the motors properly functioning.

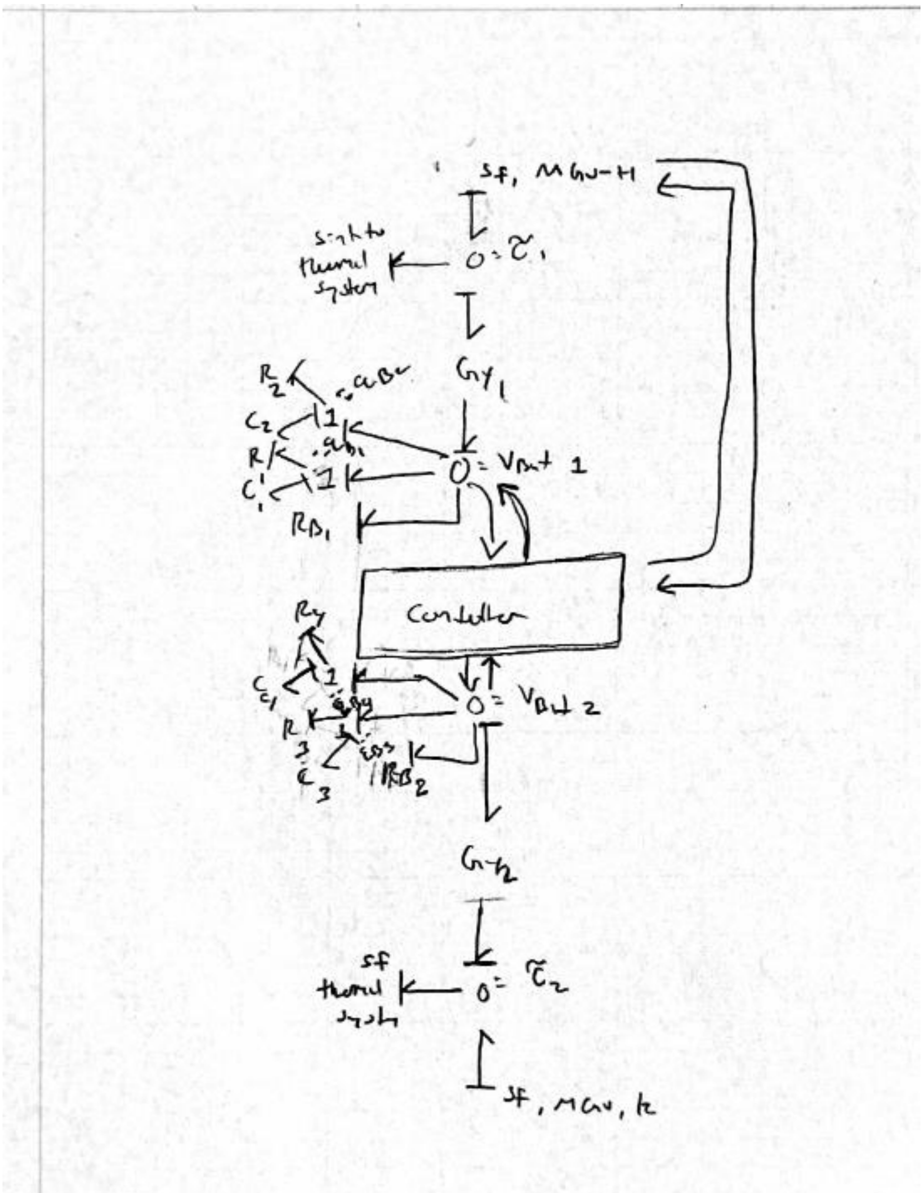


The image above illustrates the locations of the MGU-H and MGU-K with regards to the other components of an F1 car.

The image below shows a bond graph modeling the electrical components of the ERS.



A source of flow from the MGU-H enters the upper 0-junction. At this junction thermodynamic losses are subtracted, and the thermodynamic bond graph begins. A gyrator converts the MHU-H to a battery voltage. At the battery voltage 0-junction a multibank capacitor acts as the battery. A controller decides how the batteries are charged, discharged, and if the MHU-H and MGU-K are sources or sinks. Resistors are used to represent MOSFETs, which linearize the battery system and make it more accurate to real world implementations. After the battery voltage is another gyrator that converts the battery voltage to the MGU-K. This also has thermodynamic losses that are accounted for by a flow sink. At this junction is also where the unsolvable nonlinearity occurs. A solution to this nonlinearity is to separate the battery into two different batteries with information arrows to decide how each one gets used. This defeats the primary principle of the MGU-H and MGU-K sharing electrical generation and discharge but would make the model linear. This model is shown below.


Controller

The equations for this model are shown below:

$$0 = \tau_1 \left[\begin{array}{l} \tau_1 = \text{EVERYTHING} \\ S_{f,\text{MGU-H}} = S_{f,\text{thermal sink}} + \underset{}{\overset{\dot{\theta}_{GY_1}}{(GY_{1,\text{INPUT}})}} \end{array} \right.$$

$$GY_1 \left[\begin{array}{l} \tau_{GY_1} = k_{t_1} \dot{\varepsilon}_{GY_{1,1}} \\ V_{GY_1} = k_{b_1} \dot{\theta}_{GY_{1,1}} \end{array} \right.$$

$$0 = V_{BAT} \left[\begin{array}{l} V_{BAT} = \text{EVERYTHING} \\ GY_{,1,\text{OUTPUT}} = \dfrac{V_{Bat,1}}{R_{B_1}} + \varepsilon_{B_1} + \varepsilon_{B_2} \\ (\dot{\varepsilon}_{GY_1}) \end{array} \right.$$

$$1 = \varepsilon_{B_1} \left[\begin{array}{l} \varepsilon_{B_1} = \text{EVERYTHING} \\ V_{Bat_1} = \dfrac{\varepsilon_{B_1}}{C_1} + R_1 \dot{\varepsilon}_{B_1} \end{array} \right.$$

$$1 = \varepsilon_{B_2} \left[\begin{array}{l} \varepsilon_{B_2} = \text{EVERYTHING} \\ V_{Bat_2} = \dfrac{\varepsilon_{B_2}}{C_2} + R_2 \dot{\varepsilon}_{B_2} \end{array} \right.$$

$$0 : \tau_2 \left[\begin{array}{l} \tau_2 = \text{EVERYTHING} \\ S_{f,\text{MGU-K}} = S_{f,\text{thermal sink}} + \overset{\dot{\theta}_{GY,2}}{(GY_{2,\text{INPUT}})} \end{array} \right.$$

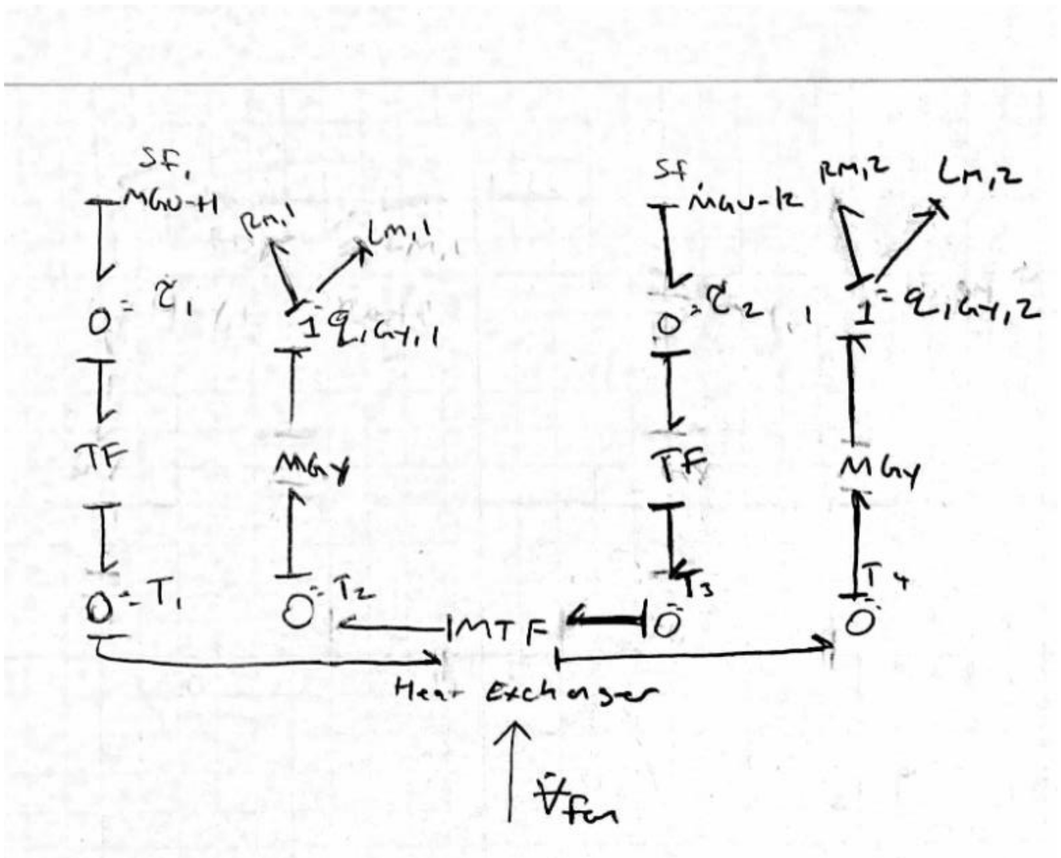
$$GY_{,2} \left[\begin{array}{l} \tau_{GY,2} = k_{t_2} \dot{\varepsilon}_{GY,2} \\ V_{GY,2} = k_{b_2} \dot{\theta}_{GY,2} \end{array} \right.$$

$$0 = V_{BAT} \left[\begin{array}{l} V_{BAT} = \text{EVERYTHING} \\ GY_{,1,\text{OUTPUT}} = \dfrac{V_{Bat,2}}{R_{B_2}} + \varepsilon_{B_3} + \varepsilon_{B_4} \\ (\dot{\varepsilon}_{GY,2}) \end{array} \right.$$

$$1 = \varepsilon_{B_3} \left[\begin{array}{l} \varepsilon_{B_3} = \text{EVERYTHING} \\ V_{Bat,2} = \dfrac{\varepsilon_{B_3}}{C_3} + \dot{\varepsilon}_{B_3} R_3 \end{array} \right.$$

$$1 = \varepsilon_{B_4} \left[\begin{array}{l} \varepsilon_{B_4} = \text{EVERYTHING} \\ V_{Bat,2} = \dfrac{\varepsilon_{B_4}}{C_4} + \dot{\varepsilon}_{B_4} R_4 \end{array} \right.$$

The bond graph for the thermodynamic ERS system is show below:



This bond graph takes in the flow sources from the previous electrical ERS bond graph that were earmarked for thermodynamic losses. Then a transformer converts the torque to a temperature. This temperature is fed through a heat exchanger and converted from a cooler temperature to resistance and inductance values for the motor gyrator element. This ensures that the performance of the motors has a relationship to temperature as these resistance and inductance values govern the efficiency of the motor. This system was already linear, so no assumptions had to be made to linearize it, except for that the fluidic aspect of the system is not important relative to the thermodynamic aspects.

The equations for the thermodynamic system for the ERS are shown below:

$0 = \tau_1 \left[\begin{array}{l} \tau_1 = \text{EVERYTHING} \\ Sf_{MGU-H} = TF_1 \end{array} \right.$

TF: $\left[TF_1 = \gamma TF_2 \right.$, where γ properly relates angular velocity to temperature

$0 = T_1 \left[\begin{array}{l} T_1 = \text{EVERYTHING} \\ TF_2 = MTF_{,1,INPUT} \end{array} \right.$

$0 = T_2 \left[\begin{array}{l} T_2 = \text{EVERYTHING} \\ MTF_{,1,output} = MGY_{1,INPUT} \end{array} \right.$

$MGY = \left[MGY_{,1,INPUT} = \gamma MGY_{,1,output} \right.$ where γ relates output temperature to current

$1 = q_{GY,1} \left[\begin{array}{l} q_{GY,1} = \text{EVERYTHING} \\ V_{GY,1} = R_{m_1} q_{,GY,1} + L_{m_1} \dot{q}_{,GY,1} \end{array} \right.$

MTF [relates the flow rate to cooling ability

$T_2 = \dot{V}_{Fn} \times \alpha \times T_1$

$T_4 = \dot{V}_{Fn} \times \beta \times T_3$

$0 = \tau_2 \left[\begin{array}{l} \tau_2 = \text{EVERYTHING} \\ Sf_{MGU-h} = TF_3 \end{array} \right.$

TF $\left[TF_3 = \gamma TF_4 \right.$, where γ proportionally relates angular velocity to temperature

$0 = T_3 \left[\begin{array}{l} T_3 = \text{EVERYTHING} \\ TF_4 = MTF_{,2,INPUT} \end{array} \right.$

$0 = T_4 \left[\begin{array}{l} T_4 = \text{EVERYTHING} \\ MTF_{,2,output} = MGY_{,2,INPUT} \end{array} \right.$

MGY $\left[MGY_{,2,INPUT} = \gamma MGY_{,2,OUTPUT} \right.$ where γ relates output temperature to current

$1 = q_{GY,2} \left[\begin{array}{l} q_{GY,2} = \text{EVERYTHING} \\ V_{GY,2} = R_{m_2} q_{,GY,2} + L_{m_2} \dot{q}_{,GY,2} \end{array} \right.$

Sources Used: Power Unit 101 with PETRONAS: MGU-H, EXPLAINED!, Power Unit 101 - Episode 4-MGU-K, *Boretti, Albert*

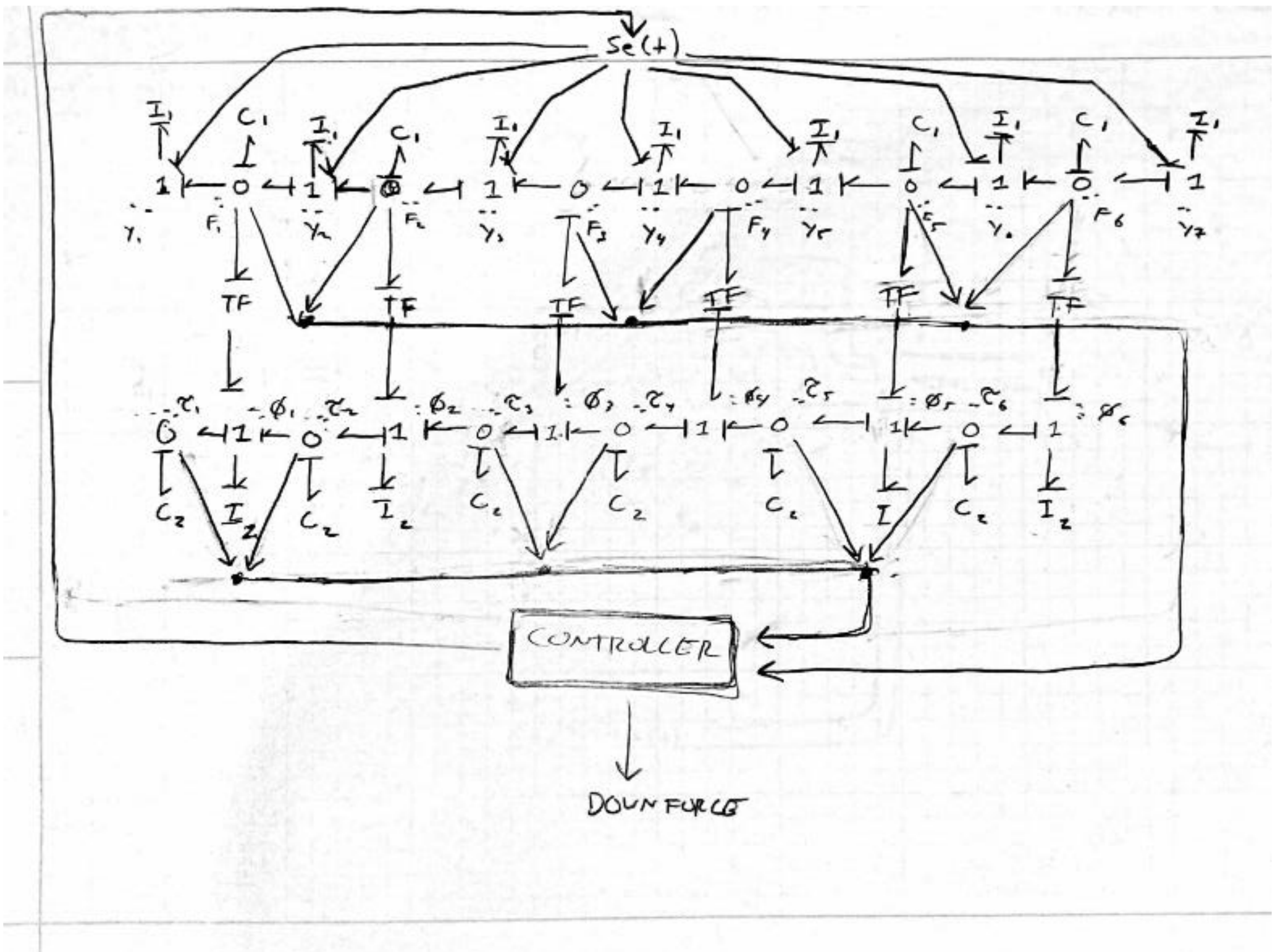
Aerodynamic Components (Front and Rear Wing)

Assumptions: The front and rear wing are assumed to be 1 airfoil with the vertical supports contributing negligible drag and downforce.

The major aspects of a F1 car when it comes to aerodynamic performance are the front and rear wings. These are the most commonly changed components at different tracks due to track locations and properties such as tight corners, and long straights. These aspects of a track change the demand for downforce, which is what this bond graph provides a linearized model for.

This image from a 1990s F1 car shows the single airfoil aspect that this bond graph models for the front and rear wing.

Below is the bond graph for the front or rear wing, since they are treated the same, except with different parameters and controller values.



This model uses 2 Timoshenko beam models, with a Euler-Bernoulli model in between. This is because the front or rear wing responds with in a uniform and continuous manner. A series of Timoshenko models is better for a system with multiple connected lumped masses, like a spine, but not an F1 aerodynamic part. The method shown above provides the best compromise which allows the Timoshenko models to accurately model the edge behavior of the airfoil while the Euler-Bernoulli model provides a continuous response for the middle of the airfoil. The way that the downforce is calculated from this model is through a controller. The controller uses speed data from the car to vary a source of effort acting at 1-junctions across the model. The controller also collects force and torque data from the 0-junctions, through information arrows. These force and torque values are used to calculate the downforce of the wing. Since this model uses existing linear bond graph models, no linearization is needed.

The equations for this model are shown below:

$$1 = Y_1 \left[\begin{array}{l} \dot{Y}_1 = \text{everything} \\ I_1 \ddot{Y}_1 = F_1 \end{array} \right.$$

$$0 = F_1 \left[\begin{array}{l} F_1 = \text{everything} \\ \dot{Y}_1 + TF_{1,1,w} + C_1 \dot{F}_1 = \dot{Y}_2 \end{array} \right.$$

$$1 = \dot{Y}_2 \left[\begin{array}{l} \dot{Y}_2 = \text{everything} \\ I_1 \ddot{Y}_2 + F_1 = F_2 \end{array} \right.$$

$$1 = \dot{Y}_3 \left[\begin{array}{l} \dot{Y}_3 = \text{everything} \\ I_1 \ddot{Y}_3 + F_2 = F_3 \end{array} \right.$$

$$0 = F_2 \left[\begin{array}{l} F_2 = \text{everything} \\ \dot{Y}_3 = TF_{2,w} + C_1 \dot{F}_2 + \dot{Y}_2 \end{array} \right.$$

$$0 = F_3 \left[\begin{array}{l} F_3 = \text{everything} \\ \dot{Y}_4 = TF_{3,w} + \dot{Y}_3 \end{array} \right.$$

$$1 = Y_4 \left[\begin{array}{l} Y_4 = \text{everything} \\ I_1 \ddot{Y}_4 + F_3 = F_4 \end{array} \right.$$

$$0 = F_4 \left[\begin{array}{l} F_4 = \text{everything} \\ \dot{Y}_5 = TF_{4,w} + \dot{Y}_4 \end{array} \right.$$

$$1 = Y_5 \left[\begin{array}{l} Y_5 = \text{everything} \\ I_1 \ddot{Y}_5 + F_4 = F_5 \end{array} \right.$$

$$0 = F_5 \left[\begin{array}{l} F_5 = \text{everything} \\ \dot{Y}_6 = TF_{5,w} + C_1 \dot{F}_5 + \dot{Y}_5 \end{array} \right.$$

$$1 = Y_6 \left[\begin{array}{l} Y_6 = \text{everything} \\ I_1 \ddot{Y}_6 + F_5 = F_6 \end{array} \right.$$

$$0 = F_6 \left[\begin{array}{l} F_6 = \text{everything} \\ \dot{Y}_7 = TF_{6,w} + C_1 \dot{F}_6 + \dot{Y}_6 \end{array} \right.$$

$$1 = Y_7 \left[\begin{array}{l} Y_7 = \text{everything} \\ I_1 \ddot{Y}_7 = F_6 \end{array} \right.$$

$$I_1 = \rho A \Delta x \qquad C_1 = \frac{kGA}{\Delta x}$$

$$\left. \begin{array}{l} TF_{1,w} = \gamma TF_{1,\omega T} \\ TF_{2,w} = \gamma TF_{2,\omega T} \\ TF_{3,w} = \gamma TF_{3,\omega T} \\ TF_{4,w} = \gamma TF_{4,\omega T} \\ TF_{5,w} = \gamma TF_{5,\omega T} \\ TF_{6,w} = \gamma TF_{6,\omega T} \end{array} \right\}$$

where γ relates linear to angular displacement

$$0 = \tau_1 \left[\begin{array}{l} \tau_1 = \text{everything} \\ C_2 \dot{\tau}_1 = \dot{\phi}_1 \end{array} \right.$$

$$1 = \phi_1 \left[\begin{array}{l} \phi_1 = \text{everything} \\ \tau_2 + TF_{1,\omega T} = \tau_1 + J_1 \ddot{\phi} \end{array} \right.$$

$$0 = \tau_2 \left[\begin{array}{l} \tau_2 = \text{everything} \\ \dot{\phi}_2 = \dot{\tau}_2 C_2 + \dot{\phi} \end{array} \right.$$

$$1 = \phi_2 \left[\begin{array}{l} \phi_2 = \text{everything} \\ \tau_3 + TF_{2,\omega T} = I_2 \ddot{\phi} + \tau_2 \end{array} \right.$$

$$0 = \tau_3 \left[\begin{array}{l} \tau_3 = \text{everything} \\ \dot{\phi}_3 = \dot{\phi}_2 + \dot{\tau}_3 C_2 \end{array} \right.$$

$$1 = \phi_3 \left[\begin{array}{l} \phi_3 = \text{everything} \\ \tau_4 + TF_{3,\omega T} = \tau_3 \end{array} \right.$$

$$0 = \tau_4 \left[\begin{array}{l} \tau_4 = \text{everything} \\ \dot{\phi}_4 = C_2 \dot{\tau}_4 + \dot{\phi}_3 \end{array} \right.$$

$$1 = \phi_4 \left[\begin{array}{l} \phi_4 = \text{everything} \\ \tau_5 + TF_{4,\omega T} = \tau_4 \end{array} \right.$$

$$0 = \tau_5 \left[\begin{array}{l} \tau_5 = \text{everything} \\ \dot{\phi}_5 = \dot{\phi}_4 + C_2 \dot{\tau}_5 \end{array} \right.$$

$$1 = \phi_5 \left[\begin{array}{l} \phi_5 = \text{everything} \\ \tau_6 + TF_{5,\omega T} = I_2 \ddot{\phi}_5 + \tau_5 \end{array} \right.$$

$$0 = \tau_6 \left[\begin{array}{l} \tau_6 = \text{everything} \\ \dot{\phi}_6 = \dot{\phi}_5 + C_2 \dot{\tau}_6 \end{array} \right.$$

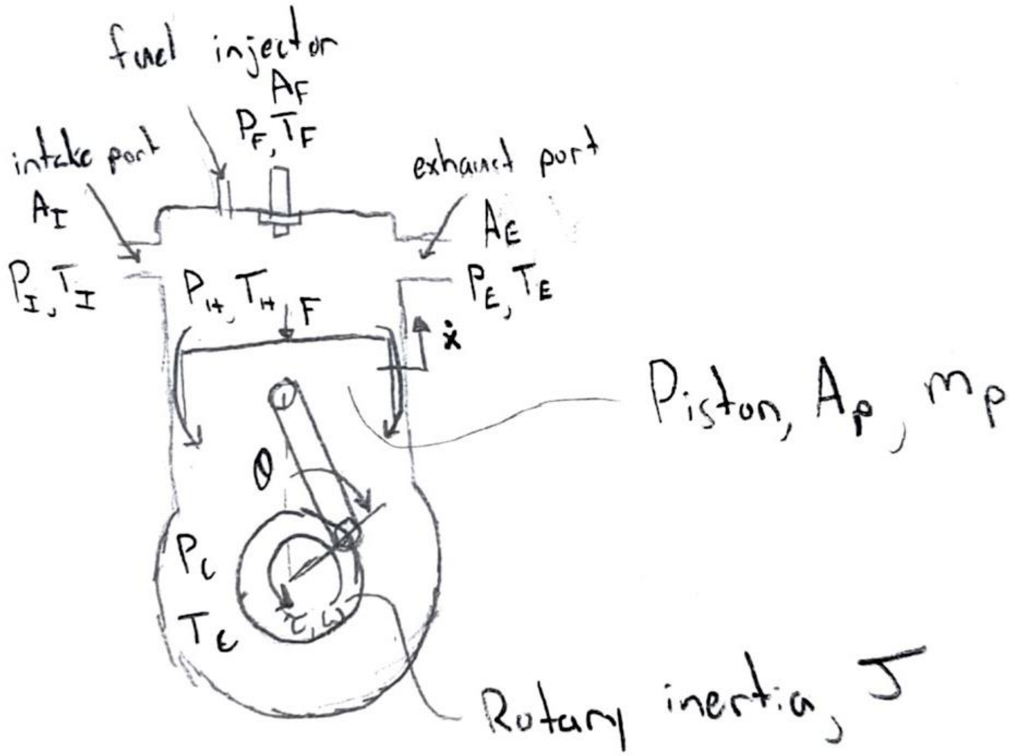
$$1 = \phi_6 \left[\begin{array}{l} \phi_6 = \text{everything} \\ TF_{6,\omega T} = I_2 \ddot{\phi}_6 + \tau_6 \end{array} \right.$$

Sources used: Karnopp, Dean C, and Ronald C Rosenberg

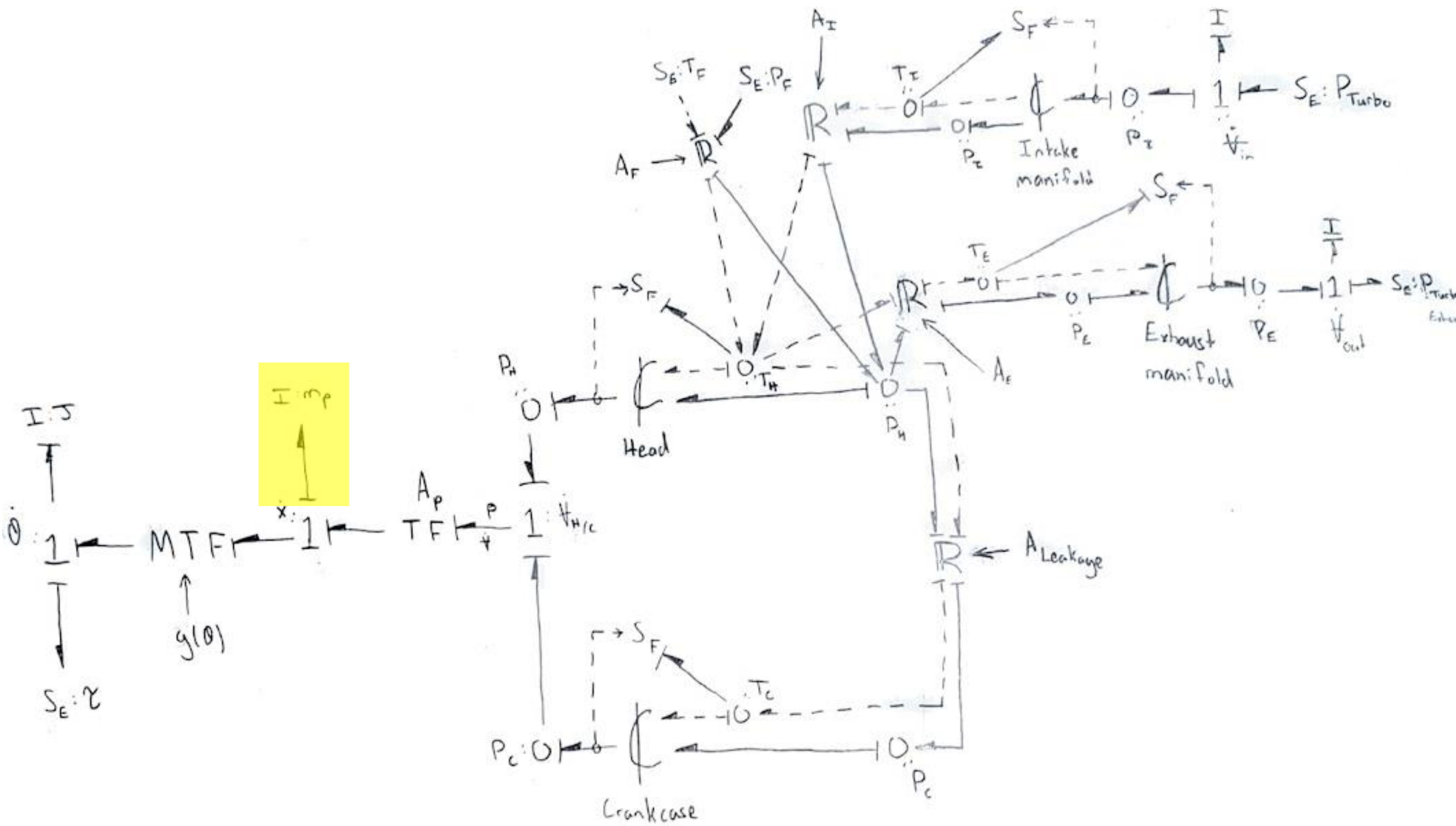
Internal Combustion Engine (ICE)

Assumptions: fuel is thoroughly atomized to the point that it can be modeled as an ideal gas, side to side motion of connecting rod mass is negligible

Modern formula 1 cars are powered by a turbocharged direct injection V6 internal combustion engine. The schematic for a single piston is shown below



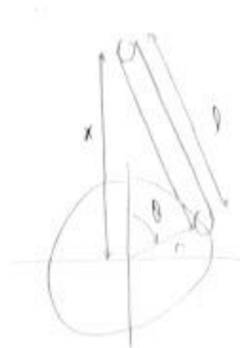
The bond graph associated with this schematic is



The P_{Turbo} is the pressure directly at the outlet of the compressor section of the turbocharger. $P_{Turbo, Exhaust}$ is the pressure directly at the inlet of the turbine section of the turbocharger. τ is the torque of the engine output shaft and flywheel. This bond graph includes the intake and exhaust manifold as well. If an equal length manifold is used, then the pressure and temperatures can be assumed to be the same at every cylinder. All inlet and outlets are modeled as isentropic nozzles. The fluid inside the head, crankcase, and the inlet/outlet manifolds are modeled as thermodynamic accumulators. A modern formula 1 engine has 6 cylinders, so each cylinder would be modeled the same and the torque outputs would be combined to effect the engine output shaft

The only nonlinearity in this model is due to the inertance associated with the mass of the piston head and connecting rod, highlighted in yellow. In order to linearize this system, you could model the piston head as massless. This would only be valid if it was engineered to be absurdly light and would likely result in model inaccuracies at higher engine RPM values. Another approach could be to add a spring/damper in between the piston head and connecting rod. This would add a zero in between the MTF and the 1 junction labeled “x_dot” with another 1 off of it which would have the R and C. This would be accurate too, since the connecting rod deflects, especially at high RPMs, due to the high forces it endures.

The equations from this bond graph are as follows:



$$1:\dot{\theta}\left\{\tau_{engine} = \tau_{rod} - J\ddot{\theta}\right.$$

$$MTF\left\{x = r\cos\theta + \sqrt{l^2 - r^2 \cdot \sin^2 A}\right.$$

$$1:\dot{x}\left\{F = m_p\ddot{x} +\right.$$

$$TF:A_p\left\{F = P_p A_p + F_{rod}\right.$$

$$1:\forall_{H/C}\left\{P_p = P_H + P_c\right.$$

$$0:P_c\left\{\dot{\forall}_{H/C} = \dot{\forall}_c\right.$$

$$C:\text{crankcase}\begin{cases} T_c = \dfrac{1}{c_v}\dfrac{E_c}{m_c} \\ P_c = \dfrac{R}{c_v}\left(\dfrac{E_c}{\forall_c}\right)\end{cases}$$

$$0:T_c\left\{\dot{E}_c = \dot{E}_{h_{Leak}} - P_c\dot{\forall}_c\right.$$

$$0:P_c\left\{\dot{m}_c = \dot{m}_{Leak}\right.$$

$$R:\text{Leakage}\begin{cases} P_r \equiv \dfrac{P_c}{P_H} \\ P_{r,crit} \equiv \left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma}{(\gamma-1)}}\end{cases}$$

if $P_r > P_{r,crit}$, then $P_r = \dfrac{P_c}{P_H}$

if $P_r \leq P_{r,crit}$, then $P_r = P_{r,crit}$

$$\dot{m}_{Leak} = A_{Leak}\frac{P_c}{\sqrt{T_c}}\sqrt{\frac{2\gamma}{R(\gamma-1)}}\sqrt{P_r^{\left(\frac{2}{\gamma}\right)} - P_r^{\left(\frac{\gamma+1}{\gamma}\right)}}$$

$$\dot{E}_{Leak} = c_p T_c \dot{m}_{Leak}$$

$$0:P_H\left\{\dot{\forall}_{H/C} = \dot{\forall}_H\right.$$

$$C:\text{Head}\begin{cases} T_H = \dfrac{1}{c_v}\dfrac{E_H}{m_H} \\ P_H = \dfrac{R}{c_v}\left(\dfrac{E_H}{\forall_H}\right)\end{cases}$$

$$0:T_H\left\{\dot{E}_H = \dot{E}_{h_F} + \dot{E}_{h_I} - \dot{E}_{h_E} - p_H\dot{\forall}_H - \dot{E}_{h_{Leak}}\right.$$

$$0:P_H\left\{\dot{m}_H = \dot{m}_F + \dot{m}_I - \dot{m}_E - \dot{m}_{Leak}\right.$$

$$R:\text{Fuel}\begin{cases} \text{if } \dfrac{P_H}{P_F} > \left(\dfrac{2}{\gamma+1}\right)^{\left(\frac{\gamma}{\gamma-1}\right)} \Rightarrow P_r = \dfrac{P_H}{P_F} \\ \text{if } \dfrac{P_H}{P_F} \leq \left(\dfrac{2}{\gamma+1}\right)^{\left(\frac{\gamma}{\gamma-1}\right)} \Rightarrow P_r = \left(\dfrac{2}{\gamma+1}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}\end{cases}$$

$$\dot{m}_F = A_F\frac{P_F}{\sqrt{T_F}}\sqrt{\frac{2\gamma}{R(\gamma-1)}}\sqrt{P_r^{\left(\frac{2}{\gamma}\right)} - P_r^{\left(\frac{\gamma+1}{\gamma}\right)}}$$

$$\dot{E}_F = c_p T_F \dot{m}_F$$

Intake:

$$\mathbb{R}: \text{Intake Valve} \begin{cases} \text{if } \frac{P_M}{P_I} > \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma}{\gamma-1}\right)} \Rightarrow P_r = \frac{P_M}{P_r} \\ \text{if } \frac{P_M}{P_I} \leq \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma}{\gamma-1}\right)} \Rightarrow P_r = \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma}{\gamma-1}\right)} \end{cases}$$

$$\dot{m}_I = A_I \frac{P_I}{\sqrt{T_I}} \sqrt{\frac{2\gamma}{R(\gamma-1)}} \sqrt{P_r^{\left(\frac{2}{\gamma}\right)} - P_r^{\left(\frac{\gamma+1}{\gamma}\right)}}$$

$$\dot{E}_I = c_p T_I \dot{m}_I$$

$$0: T_I \left\{ \dot{E}_I = E_{I,manifold} - P_I \dot{V}_I \right.$$

$$0: P_I \left\{ \dot{m}_I = \dot{m}_{I,manifold} \right.$$

$$\mathbb{C}: \text{Intake manifold} \begin{cases} T_I = \frac{1}{c_v} \frac{E_{I,manifold}}{m_I} \\ P_I = \frac{R}{c_v}\left(\frac{E_{I,manifold}}{V_{I,manifold}}\right) \end{cases}$$

$$0: P_I \left\{ \dot{V}_I = \dot{V}_{in} \right.$$

$$1: \dot{V}_{in} \left\{ P_{Turbo} = I \ddot{V}_{in} + P_I \right.$$

Exhaust:

$$\mathbb{R}: \text{Exhaust Valve} \begin{cases} \text{if } \frac{P_E}{P_M} > \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma}{\gamma-1}\right)} \Rightarrow P_r = \frac{P_E}{P_M} \\ \text{if } \frac{P_E}{P_M} \leq \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma}{\gamma+1}\right)} \Rightarrow P_r = \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma}{\gamma-1}\right)} \end{cases}$$

$$\dot{m}_E = A_E \frac{P_M}{\sqrt{T_M}} \sqrt{\frac{2\gamma}{R(\gamma-1)}} \sqrt{P_r^{\left(\frac{2}{\gamma}\right)} - P_r^{\left(\frac{\gamma+1}{\gamma}\right)}}$$

$$\dot{E}_E = c_p T_M \dot{m}_E$$

$$0: T_E \left\{ \dot{E}_E = \dot{E}_{E,manifold} - P_E \dot{V}_E \right.$$

$$0: P_E \left\{ \dot{m}_E = \dot{m}_{E,manifold} \right.$$

$$\mathbb{C}: \text{Exhaust manifold} \begin{cases} T_E = \frac{1}{c_v} \frac{E_{E,manifold}}{m_E} \\ P_E = \frac{R}{c_v}\left(\frac{E_{E,manifold}}{V_{E,manifold}}\right) \end{cases}$$

$$0: P_{E,manifold} \left\{ \dot{V}_E = \dot{V}_{out} \right.$$

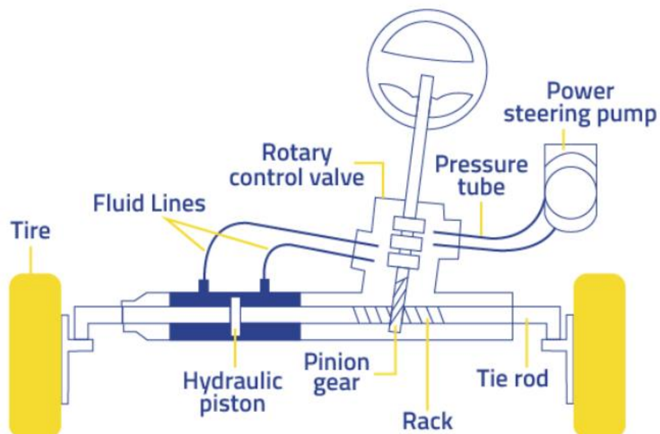
$$1: \dot{V}_{out} \left\{ P_E = I \ddot{V}_{out} + P_{Turbo,exhaust} \right.$$

Sources Used: Karnopp, Dean C, and Ronald C Rosenberg

Power Steering

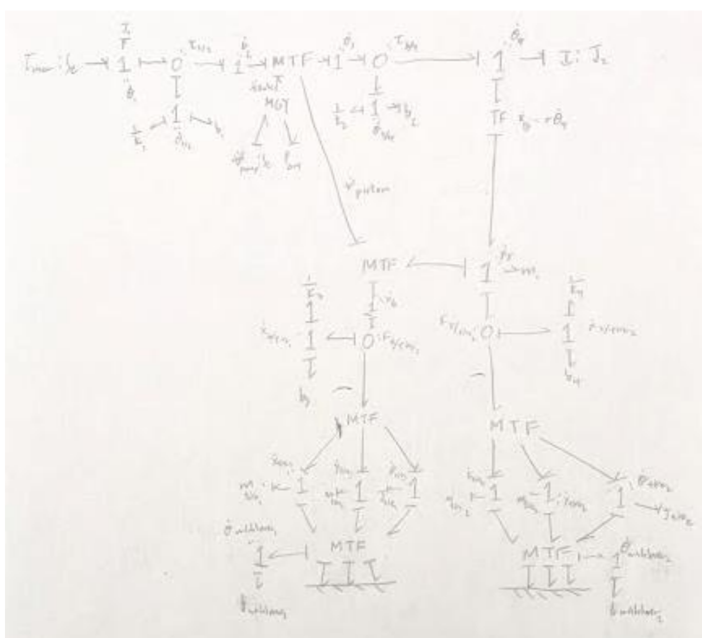
Assumptions: Parallel dampening in the rack and pinion of the power steering mechanism for linearization and use of wishbone suspension in tires.

The schematic I used to model the power steering mechanism is shown below



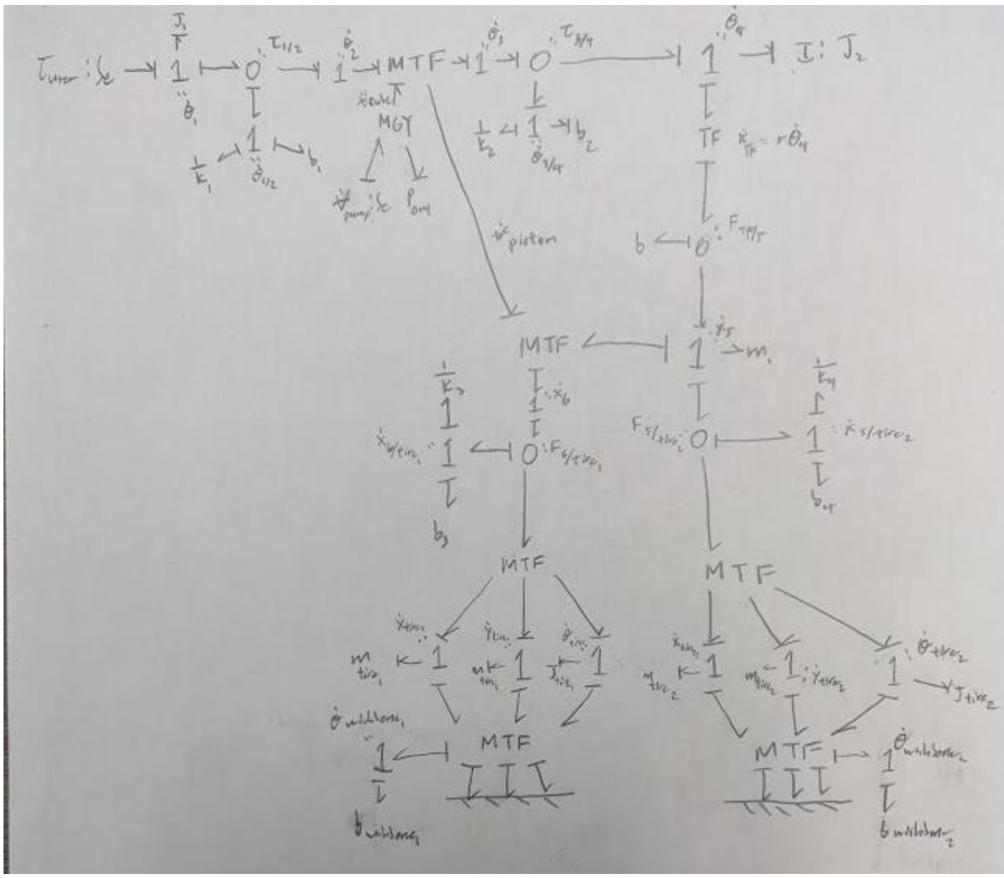
This diagram shows the power steering mechanism for a normal vehicle, some alterations as the addition of a wishbone suspension was used to liken to a Formula 1 car.

The bond graph derived from the diagram is shown below



The power steering starts with input by the driver. By turning the steering wheel, the driver applies a torque unto the main shaft, that controls the rotary control valve, allowing fluid to flow in and out of the piston and the steering pump. As the power steering pump and rotary control valve are in themselves complicated systems, we will not model the pump and model the valve as a multiport element. After the shaft connected to the steering wheel is turned, it connects to the frame of the drive base via a rack and pinion system. During the initial model, there is a single nonlinearity at the rack. In order to deal with this nonlinearity, resistance was added in parallel to the rack and pinion system. As the pinion moves and fluid is directed into the piston the length and motion of the rod creates a linkage system between the wheel and wishbone suspension system. Where the wishbone is fixed and connected to a separate part of the wheel than the lateral movement of the tie rod and the lengthening of the piston ensures that both wheels turn at the same angle.

The finalized linear bond graph of the mechanism is shown below



Sources Used: *What Is Rack and Pinion Steering?*

Additionally, the equation that correspond with this model are as follows

$$1: \dot{\theta}_1 \begin{cases} \dot{\theta}_1 = \$ \\ \tau_{motor} = J_1 \ddot{\theta}_1 + \tau_{1/2} \end{cases}$$

$$0: \tau_{1/2} \begin{cases} \tau_{1/2} = \$ \\ \dot{\theta}_1 = \dot{\theta}_{1/2} + \dot{\theta}_2 \end{cases}$$

$$1: \dot{\theta}_{1/2} \begin{cases} \dot{\theta}_{1/2} = \$ \\ \tau_{1/2} = k_1 \theta_{1/2} + b_1 \dot{\theta}_{1/2} \end{cases}$$

$$1\ \dot{\theta}_2 \begin{cases} \dot{\theta}_2 = \$ \\ \tau_{1/2} = \tau_{MTF_{in}} \end{cases}$$

$$MGY: \dot{\forall}_{pump} = \dot{\forall}_{tank} + g_1 P_{out}$$

$$MTF_1: e_1 \dot{\theta}_2 + e_2 \dot{\forall}_{tank} = \dot{\theta}_3$$

$$1: \dot{\theta}_3 \begin{cases} \dot{\theta}_3 = \$ \\ \tau_{MTF_{out}} = \tau_{3/4} \end{cases}$$

$$0: \tau_{3/4} \begin{cases} \tau_{3/4} = \$ \\ \dot{\theta}_3 = \dot{\theta}_{3/4} + \dot{\theta}_4 \end{cases}$$

$$1: \dot{\theta}_{3/4} \begin{cases} \dot{\theta}_{3/4} = \$ \\ \tau_{3/4} = k_2 \theta_{3/4} + b_2 \dot{\theta}_{3/4} \end{cases}$$

$$1: \dot{\theta}_4 \begin{cases} \dot{\theta}_4 = \$ \\ \tau_{3/4} = J_2 \ddot{\theta}_4 + \tau_{TF} \end{cases}$$

$$TF: \dot{x}_{TF} = r \dot{\theta}_4$$

$$0: F_{TF/5} \begin{cases} F_{TF/5} = b \dot{\theta}_{TF/5} \\ \dot{x}_{TF} = \dot{x}_{TF/5} + \dot{x}_5 \end{cases}$$

$$1: \dot{x}_5 \begin{cases} \dot{x}_5 = \$ \\ F_{TF/5} = m_1 \ddot{x}_5 + F_{5/tire_2} + F_{MTF_{2in}} \end{cases}$$

$$0: F_{5/tire_2} \begin{cases} F_{5/tire_2} = \$ \\ \dot{x}_5 = \dot{x}_{5/tire_2} + \dot{x}_{MTF_3} \end{cases}$$

$$1: \dot{x}_{5/tire_2} \begin{cases} \dot{x}_{5/tire_2} = \$ \\ F_{5/tire_2} = k_4 x_{5/tire_2} + b \dot{x}_{5/tire_2} \end{cases}$$

$$MTF_3: \dot{x}_{tire_2} e_3 + \dot{y}_{tire_2} e_4 + \dot{\theta}_{tire_2} e_5 = \dot{x}_{MTF_3}$$

$$MTF_4: \dot{\theta}_{wishbone_2} = e_6 \dot{x}_{tire_2} + e_7 \dot{y}_{tire_2} + e_8 \dot{\theta}_{tire_2}$$

$$MTF_2: e_9 \dot{x}_5 + e_{10} \dot{\forall}_{piston} = \dot{x}_6$$

$$1: \dot{x}_6 \begin{cases} \dot{x}_6 = \$ \\ F_{MTF_2} = F_{6/tire_1} \end{cases}$$

$$0: F_{6/tire_1} \begin{cases} F_{6/tire_1} = \$ \\ \dot{x}_6 = \dot{x}_{6/tire_1} + \dot{x}_{MTF_5} \end{cases}$$

$$1: \dot{x}_{6/tire_1} \begin{cases} \dot{x}_{6/tire_1} = \$ \\ F_{6/tire_1} = k_3 x_{6/tire_1} + b_3 \dot{x}_{6/tire_1} \end{cases}$$

$$MTF_3: \dot{x}_{tire_1} e_{11} + \dot{y}_{tire_1} e_{12} + \dot{\theta}_{tire_1} e_{13} = \dot{x}_{MTF_5}$$

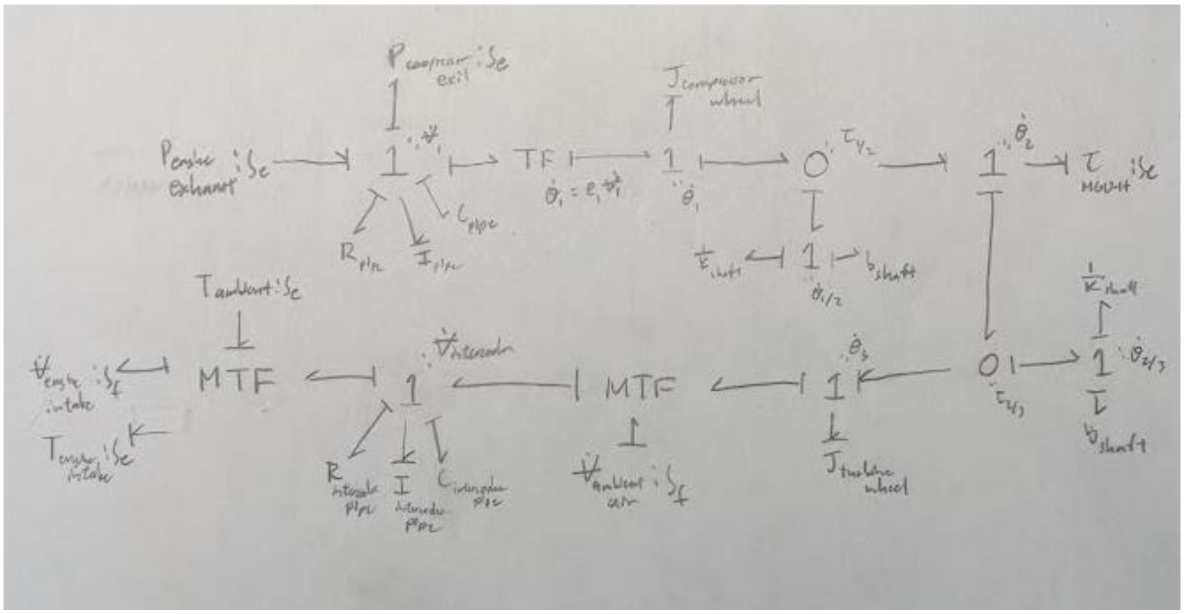
$$MTF_4: \dot{\theta}_{wishbone_1} = e_{14} \dot{x}_{tire_1} + e_{15} \dot{y}_{tire_1} + e_{16} \dot{\theta}_{tire_1}$$

Due to the number of multiport elements and transformers, the equations are left with unknown constants.

Turbo

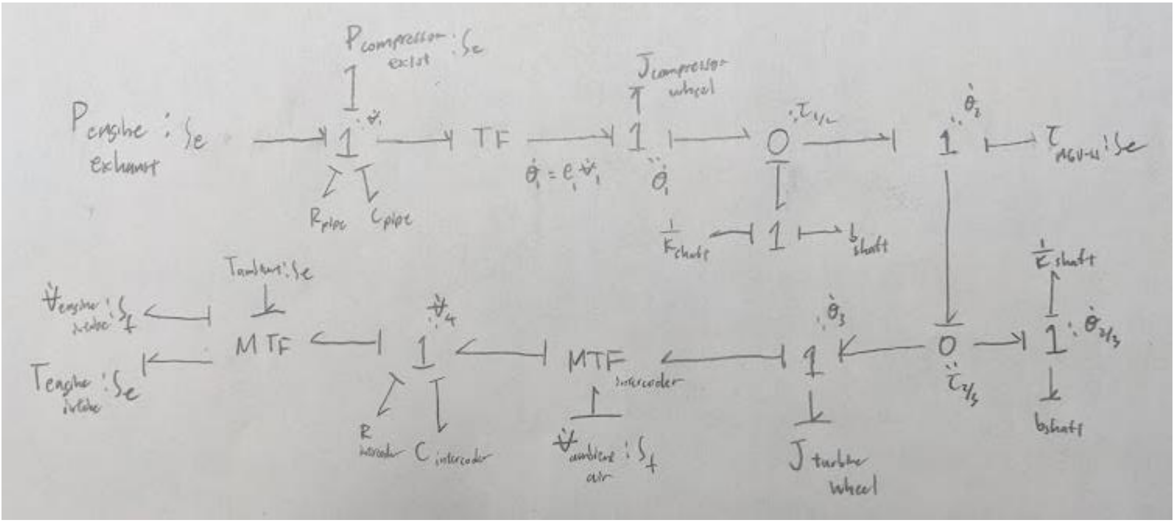
Assumptions: We are assuming that since this is an engine for an Formula 1 race car, that the pipes are as short as possible for fitting. Additionally, we are modelling the initial states of turning on the turbo and engine to the rotation of the compressor and turbine is solely from the exhaust of the engine and is not already spinning.

The schematic used for reference in creating our bond graph is the same as the diagram used to model the energy recovery system. The initial bond graph created from that model is as shown

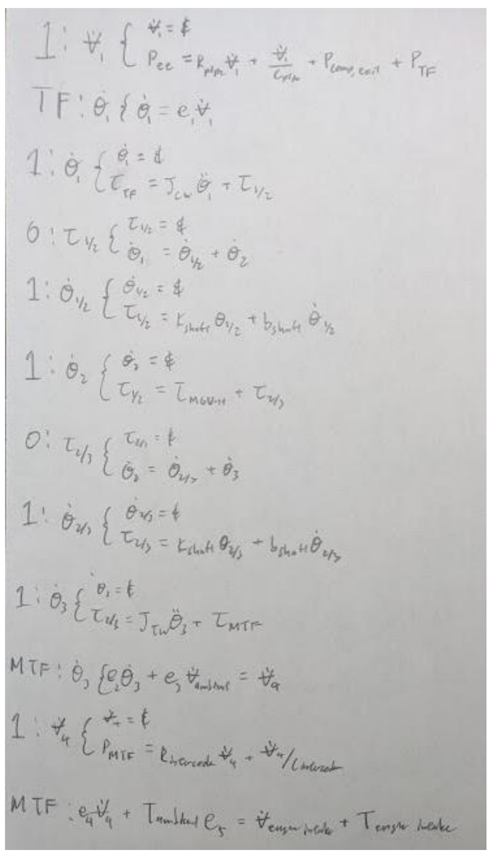


The turbo is located after the engine, so the exhaust of the engine is actually the intake into the compressor stage, the first stage of a turbo. After going through a pipe, the exhaust turns the compressor wheel and then exits the compressor stage. While the compressor wheel is turning, part of the rotational velocity is give energy into the car MGU-H, motor generator unit – heat, and the rest goes to spin the turbine wheel. As the turbine wheel spins, the turbine attracts ambient air and transports that air into the intercooler so the air can cool and become more dense. This denser air is then transferred into the intake of the engine as the denser air makes the engine more efficient and powerful.

The nonlinearities found in the original bond are due to any impedance caused by the pipes. Given the that pipes of an engine and fairly large in diameter and short, we've determined that the impedance is negligible. This linearized bond graph is as follows



The equations derived from the finalized bond graph are shown below



Due to the complexity of the multiport elements, the equations for the element are created by inputting constants in front of the variables.

Sources Used: *Boretti, Albert*

General Wrap-up

Our biggest takeaway from this project is a much deeper understanding of how each of these subsystems work. Most of us had a high-level understanding of the operating principles of these components but making valid models required us to learn how they work on a low-level. For example: the concept of car suspension is easily understood, but the concept of sprung vs un-sprung mass is something that some of us weren't familiar with. These are the sorts of things that require a system model to fully appreciate the impact of. We were also able to learn how these differing components interact and work with each other. For example: we knew that the MGU was an energy recovery system, but we didn't fully understand how the MGU-H gets the energy from the turbo.

Our other big takeaway is a level of comfort in fully developing system models ourselves. In previous Rose-Hulman courses, such as ADES, we've often been given a nice schematic to base a system model off of. For this project, we had to start from nothing and find any relevant info from textbooks or research papers. This has made us much more confident in our ability to apply modeling skills to real world situations.

One value of this class is our improved understanding of how to model multi-domain systems. As MEs/BEs, we were already comfortable with mechanical systems and basic electrical systems, but anything else was daunting. Different domains were modeled with their own unique techniques and theories. After having learned bond-graph, these types of systems are much more approachable. In addition, we learned that we do not need to be experts in any specific domain in order to model it effectively or be able to understand how it changes a system as a whole. Another great value from this class is the newfound speed at which we can model engineering systems. We all know how to use COLM/COAM to model mechanical systems, as taught in MSYS, but bond-graph is much faster and easier.

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